These learning activities are designed to facilitate students' development of spatial reasoning skills through the use of attribute blocks.

Linking the Van Hiele Theory to Instruction

By Tashana D. Howse and Mark E. Howse

Geometry is the branch of mathematics that addresses spatial sense and geometric reasoning. Students begin to understand geometry through direct interaction with their physical world. Because it is the study of the physical attributes of the environment, geometry has relevance for every student; the world becomes a big classroom. As students see, touch, and manipulate shapes, they begin to develop spatial reasoning skills.

Although geometry is an integral part of the curriculum, many students fail to develop a deep understanding of basic geometric concepts (O'Brien 1999; Clements 2003). According to the National Council of Teachers of Mathematics (NCTM 2000), K–grade 12 geometry instruction should empower students with the ability to analyze properties of geometric shapes and to base sound arguments on the understanding of relationships among these properties. Kindergarten students should be able to "describe attributes and parts of two-dimensional shapes" (NCTM 2006, p. 24). Geometry instruction at the primary level is best facilitated using concrete models, drawings, and dynamic software as appropriate. This article will discuss activities that are designed to use attribute blocks to facilitate students' development of spatial reasoning skills. The Van Hiele theory offers a framework for reflecting on these activities and their instructional value in fostering deep geometric understandings for students.

TABLE 1

The Van Hiele theory of geometric thought describes the different levels of understanding through which students progress when learning geometry.

Van Hiele theory of geometric thought

Level	Description	Ability of student
1	Visual	Describes shapes on the basis of their appearance
2	Analysis	Describes shapes on the basis of their properties
3	Abstraction	Recognizes the importance of properties and the relationships among them, which assist students in logically ordering the properties of the shapes
4	Deduction	Attains logical reasoning ability and proves theorems deductively
5	Rigor	Establishes and analyzes theorems in different postulation systems

TABLE 2

Five phases of learning support students as they progress through levels of geometric thought.

Framework of the Van Hiele phases of learning

Phase	Descriptions
Information	Students develop vocabulary and concepts for a particular task. The teacher assesses students' interpretation/reasoning and determines how to move forward with future tasks.
Directed orientation	Students actively engage in teacher-directed tasks. They work with the developments from the previous stage to gain an understanding of them as well as the connections among them.
Explication	Students are given the opportunity to verbalize their understanding. The teacher leads the discussion.
Free orientation	Students are challenged with tasks that are more complex and discover their own ways of completing each task.
Integration	Students summarize what they have learned, creating an overview of the concept at hand.

Overview of the Van Hiele model

The Van Hiele theory of geometric thought describes the different levels of understanding through which students progress when learning geometry (Van Hiele 1984). The basis of the theory is the idea that a student's growth in geometry takes place in terms of distinguishable levels of thinking. Geometry instruction should be designed with these levels in mind (Choi-koh 1999). The Van Hiele model for the theory of geometric thought consists of five levels (see **table 1**) (Burger and Shaughnessy 1986; Clements 2003):

1. Visualization
2. Analysis
3. Abstraction
4. Deduction
5. Rigor

The Van Hieles asserted that students must develop masterfully at each level before they are able to progress to the next: "These levels are sequential, invariant, and hierarchical" (Clements 2003, p. 152). Progression from one level to the next is best facilitated through strategically planned instruction (Van Hiele-Geldof 1984; Clements 2003). During early elementary school, students tend to move from level 1 to level 2. For example, at level 1, students name all shapes that look like boxes as *rectangles*. Then at level 2, students sharpen the ability to describe attributes, such as characterizing a rectangle as a flat shape with four sides and four corners. Also at level 2, the rectangle is described as a *quadrilateral* with four sides, four right angles, and with congruent and parallel opposite sides. The Van Hieles suggested that students develop a deep understanding of all geometric concepts through a similar progression from level 1 to level 5. Thus, effective geometry teaching guides students through each of these levels. According to the Van Hiele model, the five phases support students as they progress through the levels of geometric thought (see **table 2**). In this article, we describe a sequence of activities that we developed to capitalize on and enhance elementary school students' geometric thinking using this five-phase framework.

Using the learning phases

The geometry lesson started with the use of attribute blocks to help a class of young students sharpen their spatial reasoning skills. The class consisted of twenty kindergarten students whom we divided into five groups of four. Each group received a set of attribute blocks. The lesson took place in the middle of the school year, and the students had prior experience with sorting different objects and working with basic geometric shapes, such as squares, rectangles, circles, and triangles. The students had also worked with Venn diagrams before the lesson. Their experience with Venn diagrams, however, was limited to words and not shapes, a task that was included in the lesson that we will discuss later in this article. The goal of these activities was to have students reason with the physical properties and characteristics of the attribute blocks, which is the visual level, level 1, of the Van Hiele levels of geometric thought.

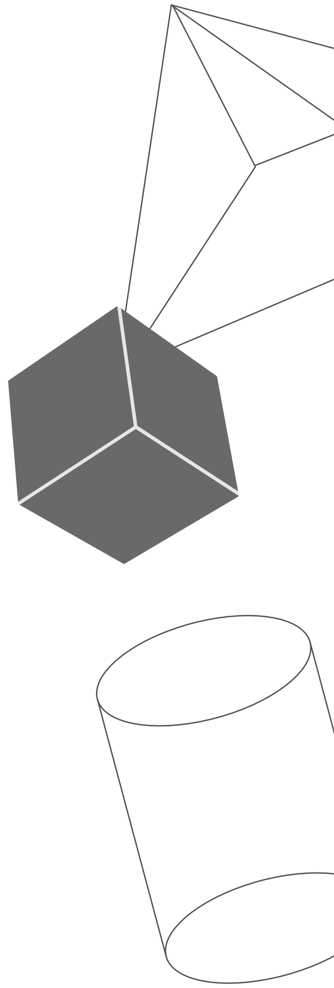
The information phase

According to the Van Hieles, this first phase includes dialogue between the teacher and students regarding the geometric objects of focus. The purpose of this dialogue is primarily to inquire into students' prior knowledge of geometric shapes or concepts as well as to lay a foundation for subsequent learning activities (Van Hiele-Geldof 1984). To begin instruction on the geometry lesson, we wanted to assess students' prior knowledge while helping them develop vocabulary specific to the attribute blocks. Therefore, we used two sorting activities to assist students in exploring general characteristics of basic geometric shapes. The objective of the initial sorting activities was to help students identify all the attributes of the blocks: color, shape, size, number of sides, and whether the shape had corners. First, we directed students to sort the blocks into two or more categories on the basis of any set of shared attributes. Three of the five student groups sorted the blocks by both color and shape. These are the most distinctive attributes of the blocks that students tend to focus on. One group, however, sorted the blocks by the number of sides. Another group sorted by shapes with and without corners. Students from each group were asked to explain the rationale for their sort. We were particularly interested in the groups that did not sort on the basis of color and shape. The group that sorted by number of sides made the following statements:

- "Shapes with six sides are hexagons."
- "Shapes with four sides are squares and rectangles."
- "Shapes with three sides are triangles."
- "Shapes with no sides are circles."

The students who sorted by corners also shared their sorting strategy with the class. Allowing these groups to communicate their reasoning about the blocks enabled the other groups to see alternative ways of sorting, which gave them a different perspective on the blocks. NCTM's Communication Process Standards support this strategy (NCTM 2000). As a class, we discussed the number of categories that would be associated with sorting on the basis of either corners or number of sides. The class concluded that four categories would result for the number of sides and two categories for the other way. To assess understanding, we instructed the groups to replicate the sorts. During this part of the lesson, we were able to gauge students' prior knowledge; they were knowledgeable about the most distinctive characteristics of the blocks: color and type of shape. Additionally, they were knowledgeable about specific vocabulary terms: *sides* and *corners*. This information assisted us in transitioning into the next sorting activity, which was for students to identify the remainder of block characteristics.

Neither of the groups considered sorting the attribute blocks on the basis of relative size or



Van Hiele phases of learning

Van Hiele-Geldof (1984) suggested that teachers, when planning geometry instruction, use the five Van Hiele phases of learning:

1. Information
2. Directed orientation
3. Explication
4. Free orientation
5. Integration

thickness. So, to further extend their geometric thinking, two blocks were displayed: a large, thick, red circle and a small, thin, yellow circle. Then, we posed the following question, "Can someone tell me the difference between these two shapes?"

A female student stated, "One is big, and one is small."

A male student offered, "One is red, and one is yellow."

These correct statements encouraged a third observation: "One is fat, and one is skinny." We referred to these shapes as *thick* and *thin*.

To clarify students' understanding, the class was instructed to find all the thin shapes and then all the big shapes. Sharing and discussion made clear that most students could identify all the characteristics of each shape.

Directed orientation

On the second day of the geometry lesson, the class reviewed the attributes that we had discovered by sorting the blocks into categories the day before (see **fig. 1**). After the review, students engaged in a series of activities that are consistent with the second Van Hiele phase of learning, the directed-orientation phase. Directed orientation is characterized by structured activities that challenge students to formally recognize and verbalize their understanding of the new geometric concepts that were introduced in the information phase (Van Hiele-Geldof 1984). Through teacher-directed activities, students should gain an understanding of the attributes and the connections among them. For the first learning activity, students were required to categorize the attribute blocks into sets of two with one attribute difference (see **fig. 2**).

The objective of the first activity was to deepen students' understanding of the basic shapes by encouraging students to make meaningful connections among the various attributes. To begin, we held up two triangles: a big, thick, blue triangle and a small, thick, blue triangle. Students were to examine these shapes and describe their similarities. Raising a hand to answer, students were able to describe that both shapes were blue, thick, had three sides and three corners, and were triangles. However, one student in particular

FIGURE 1

On day 2, the class reviewed the attributes that students had discovered the previous day when sorting blocks into categories.

Characteristics of attribute blocks

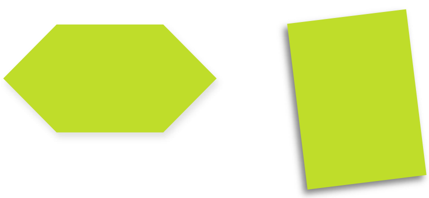
1. Small—Large (Big)
2. Thin—Thick
3. Red—Blue—Yellow
4. Triangle (3 sides)
   Hexagon (6 sides)
   Square (4 sides)
   Circle (no sides)
   Rectangle (4 sides)
5. With corners—Without corners
   (sides) (no sides)

FIGURE 2

Students categorized blocks into sets of two—with one attribute difference, two differences, and finally three—the goal being for them to gain an understanding of attributes and connections among them.

Difference: Color

Difference: Shape–thickness



Difference: Color–shape–thickness

shouted out, "One shape is big, and one shape is small." This led the class to discuss the distinction between *similarities* and *differences* among the attributes of the blocks. The class agreed that the last statement referred to the difference between the two triangles. These triangles contained one difference: one was big, and the other was small. This discovery led us into the next activity for this phase.

Every student was instructed to find two blocks that contained only one difference. After finding the shapes, students held their blocks up. They were instructed to describe the difference between the pair of blocks that they chose. As students shared their descriptions, other students listened to ensure that their findings were correct. Some findings included the following:

- "I have two squares, but they are different colors."
- "I have two circles, but one is skinny, and one is fat."
- "I have a small square and a small circle."

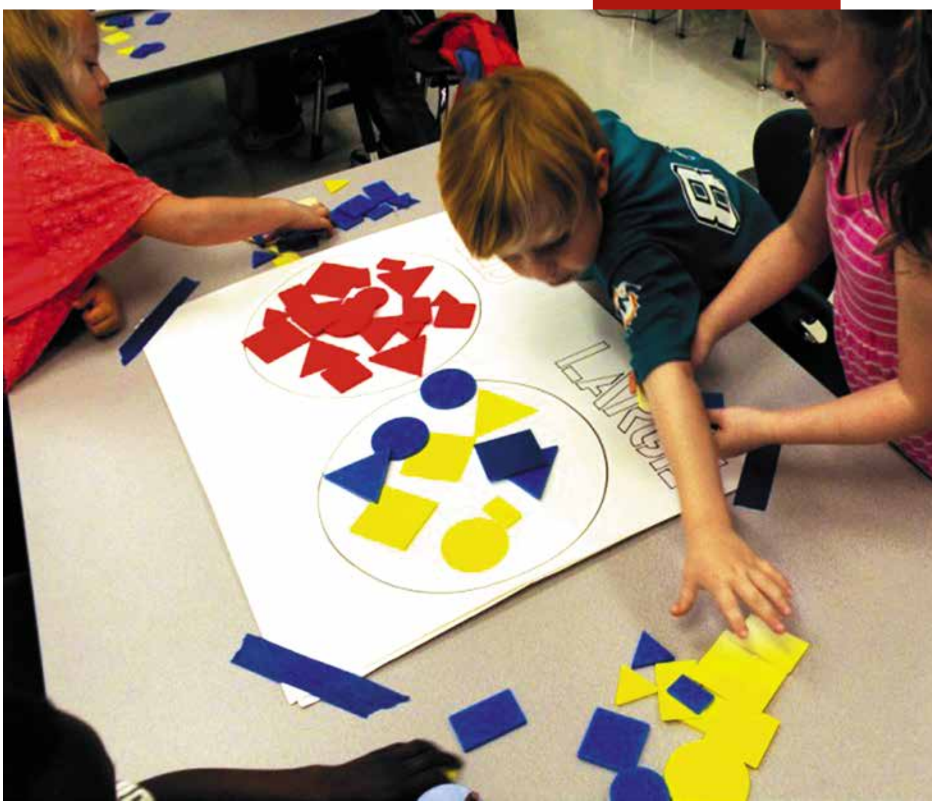
As students demonstrated mastery of finding sets of shapes with one attribute difference, they were encouraged to find shapes with two attribute differences, and then three attribute differences. At times, students interpreted a presenter's finding by guessing on the basis of given sets. For example, if a student held up one big, thick, blue circle and one big, thin, blue triangle, then how many differences exist? Yes, there are two differences: in shape and thickness. When a student held up two shapes that he or she thought contained two differences, the student was reasoning among the attributes. Students within the same group critiqued the reasoning of that student, determining if he or she was correct. As an example of such communication, one student in particular held up a small, thick, red triangle and a large, thick, blue circle. Another student stated, "Those shapes have three differences instead of two." As a result, the student holding up the shapes reflected on his thinking and decided to pick up a small, thick, red triangle and a small, thick, blue circle.

This interaction among students exemplifies the reasoning and sense making described by NCTM as well as the practices that promote mathematically proficient students as described by the Standards of Mathematical Practice (SMP) in the Common Core State Standards (CCSSI 2010) and by the National Research Council (NRC 2001).

Grouping by three attribute differences was difficult for most of the students. Only one group was successful. When asked to describe the group's strategy, one of its members explained, "First I see how the shapes are alike."

Many of the students focused on similarities among the shapes before identifying differences. The strategy of working backward assisted students in finding shapes that fit this category. For example, referring to the previous dialogue, the student held up *a small, thick, red triangle* and *a large, thick, blue circle.* A student who works backward would identify how the shapes are alike first: they are both *thick*. Therefore, everything else is a difference: color, shape, and size. In other words, if the shapes have only one similarity, then they have three differences. Sharing this student's finding with the class prompted the other groups to continue searching for shapes

The authors strategically introduced Venn diagrams, first describing labels on two empty, nonintersecting circles. Then student groups carefully completed the diagram. The goal of using Venn diagrams was to have students focus their dialogue on the attributes of the blocks.



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Although using Venn diagrams challenged the kindergartners, by working in groups and helping one another, they quickly learned to order attributes logically.

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with three attribute differences. The student responses to this activity were indicative of their abilities to recognize and verbalize their understanding among the characteristics and properties of the shapes. As a result, they could make connections among the properties and characteristics, which is consistent with the Van Hiele directed-orientation phase.

Explication

According to Van Hiele, the explication phase of learning involves engaging students in verbalizing their understanding of the geometric concepts that they have observed. The role of the teacher during this phase is to facilitate dialogue that allows students to explain their understanding using the appropriate language. The specific goal of the activity for this phase was to have students use Venn diagrams as a focus of dialogue regarding the attribute blocks. To begin this exercise, students received specific sets to use. Venn diagrams were strategically introduced to students by starting with two empty, nonintersecting circles. Before students began, we described the labels on the circles. Then the groups were instructed to carefully complete the diagram. First, they were to place all the red blocks in the appropriate circle. Once they completed this task, they were instructed to place all the large/big shapes in the other circle. We observed that students became puzzled about where to place the small shapes, which were all yellow and blue. Some students would ask, "Where do these go?" Other students would state that those shapes did not belong in the circles. The entire class addressed the following questions, forcing students to justify their reasoning among the attributes of the shapes: Are there any shapes that do not belong? Why?

In their small groups, students placed all the large, red shapes in the intersection between the *red* and *large* circles of a Venn diagram.

With both circles filled, the groups were directed to closely observe the shapes that were in the circle that was labeled blue. "What do you notice about the shapes in this circle?" we asked. Student responses included the following:

- "They are thick and thin."
- "They are circles, squares, and rectangles."
- "They are large and small."

The last statement prompted the next question: "If we have both large and small shapes in the blue circle, then what does this indicate for us?"

A student replied, "The big, red shapes can also go in other circles."

While students were thinking about this statement, we replaced two nonintersecting circles with two interlocking circles. Then we asked the class, "Now that the circles are together, which shapes will go in the middle?"

A student replied, "The big, red shapes will go in the middle."

The groups were instructed to place all the large, red shapes in the intersection between the red and larges circles of the Venn diagram; students later placed the rest of the circles accordingly.

At the end of this lesson, each group received another Venn diagram. Within their groups, students were instructed to complete the new Venn diagram. We walked around the room to assess students' understanding by observing and asking questions. The activity assisted students in understanding how Venn diagrams work. For some students, the task was challenging; but working in groups enabled them to assist one another. Students were beginning to order attributes logically. They could verbalize their understandings of the attributes of the shapes through connections, a skill that is consistent with the Van Hiele explication phase of learning.

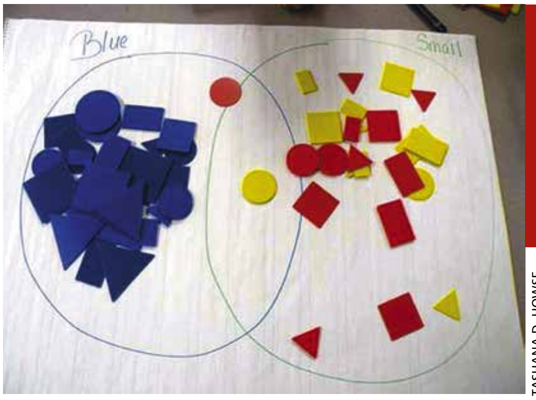
Free orientation

After reflecting on students' understanding of the attributes and their understanding of a Venn diagram, we thought it would be useful to allow students to create their own Venn diagram situations. So, on the fourth day of the lesson, we requested that every group complete a peer-made Venn diagram to ensure their understanding of the relationships and attributes of the blocks. This activity is consistent with the free orientation phase, in which students develop their own way to complete geometric tasks. To make nice circles, students traced a circle that we supplied for them. Examples of peer-made Venn diagrams included (1) blue and small; (2) thick and yellow; and (3) thin and rectangle. However, students were not yet at the level to write these types of words, so they abbreviated the words by using the first letter of the word.

The next task allowed students to use their creativity. Individual students began to create Venn diagrams by drawing shapes within the circles. Students really enjoyed this task; they drew, colored, and demonstrated their understanding of the concept of a Venn diagram. As students worked individually, we assessed their understanding by asking questions about their drawings. Some of our questions included the following:

1. Since one circle represents blue shapes and the other circle represents purple shapes, what type of shapes would go in the middle?
2. Do you have any small shapes in the blue circle? Could you also consider them as small? What would go in the middle?
3. How are the shapes in the middle different from the outer shapes?

While assessing them, we reminded students of the previous activity in the explication phase. This Venn diagram activity enhanced our students' ability to articulate their understanding of the related geometric concepts. It allowed them to critically think about the attributes that they had learned individually.



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Individual students used their creativity, also drawing shapes within circles to demonstrate their understanding of the concept of a Venn diagram.

Integration

To review the geometric concepts developed through this series of activities and to complete the five phases of learning, we progressed to the last learning phase, integration. In this phase, students use all the concepts to complete a task. During the culminating activity, students played a game that is similar to Scrabble®. Instead of having spaces for letters, the game board is a Venn diagram with two or three intersecting circles (see **fig. 3**). Instead of blindly choosing seven letters, students blindly chose seven attribute blocks from a bag. The

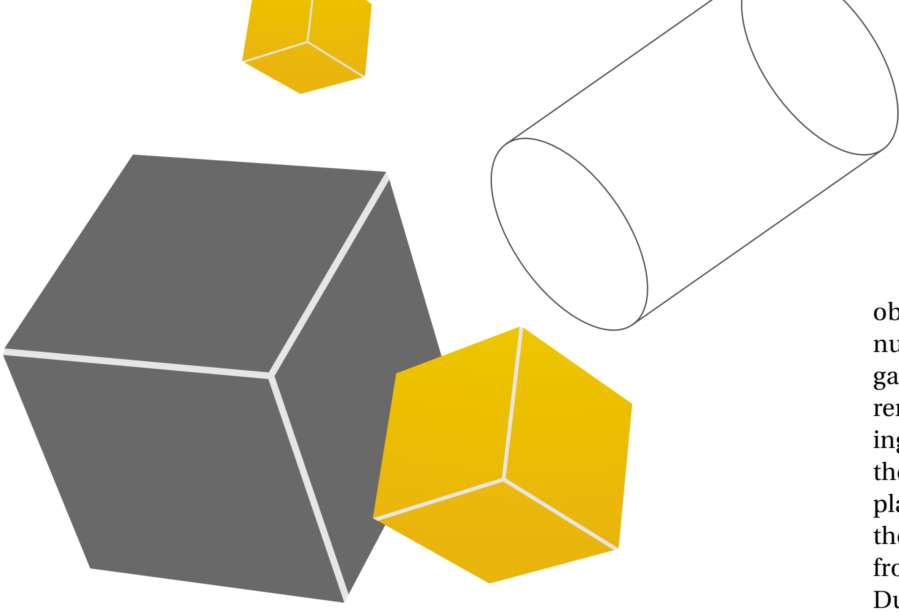

FIGURE 3

During the culminating activity of the geometry lesson, students played a game that was similar to Scrabble®, but instead of a game board with spaces for letters, they used a Venn diagram with two or three intersecting circles.

(a) Game board **(b) Advanced game board**

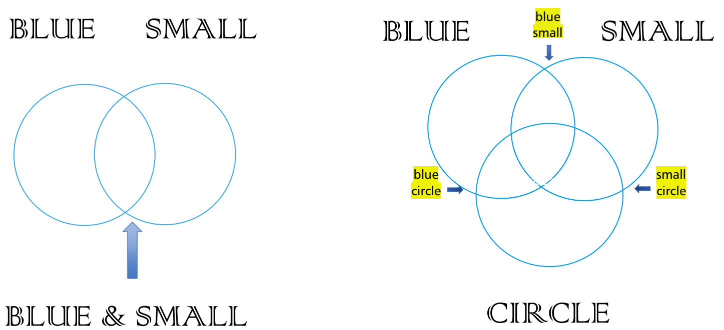

TABLE 3

The authors created an observational checklist to assess learning during the game. The goal for students was to reason and interpret the related concepts with confidence and accuracy.

Observational checklist behaviors
❑ Student demonstrates the ability to reason among the attributes of the blocks.
❑ Student demonstrates the appropriate interpretation of a Venn diagram.
❑ Student successfully places appropriate shapes in the appropriate location on the Venn diagram.
❑ Student exudes confidence when placing blocks on the Venn diagram.
❑ Student recognizes that certain blocks may not belong on the Venn diagram.

objective of the game is to have the fewest number of shapes left in your possession at game's end. Shapes that do not belong will remain in students' possession. Before starting the game, we clarified the descriptions of the circles for each group. Students took turns placing a block in the appropriate section on the Venn diagram and removing another block from the bag until all the blocks were gone. During this activity, students reasoned with the attributes of the blocks while critiquing the reasoning of others (CCSSI 2010). This game allowed students to reflect both internally and externally on the relationships among the attributes of the blocks. Although the game was a little challenging for some students, we observed learning taking place (see **table 3**).

Sharing perspectives

The Van Hiele theory of geometrical thought and phases of learning provide a framework for effective geometry instruction. Activities like those described in this article draw on well-structured instruction and purposeful interaction among students to promote deep understanding of geometric concepts. The set of activities not only benefited students but also enhanced our skills as mathematics educators. We were successful in engaging students in activities that are consistent with Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010). During certain stages of the activities, students were prompted to share their ways of reasoning by constructing viable arguments in relation to the attribute blocks. Additionally, students had opportunities to not only reflect on their own thinking but also critique the reasoning of their classmates. These student practices are consistent with the Standards for Mathematical Practice (SMP), specifically SMP 3, which promotes classroom discourse during which students can engage in mathematical talk (CCSSI 2010; Cobb and Yackel 1995). For students to do this, teachers must provide students with opportunities to reason with mathematics as well as opportunities to communicate their understandings to their classmates. Through well-planned instruction, teachers can facilitate students' engagement in this practice.

Throughout the series of activities described in this article, students were prompted to share their findings, allowing their classmates to see and critique other perspectives. Use of the five-phase framework assisted us in designing appropriate instruction to enhance students' spatial reasoning while promoting communication about mathematics in the classroom.

Common Core Connections
K.G.2
K.G.4
K.MD.3

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